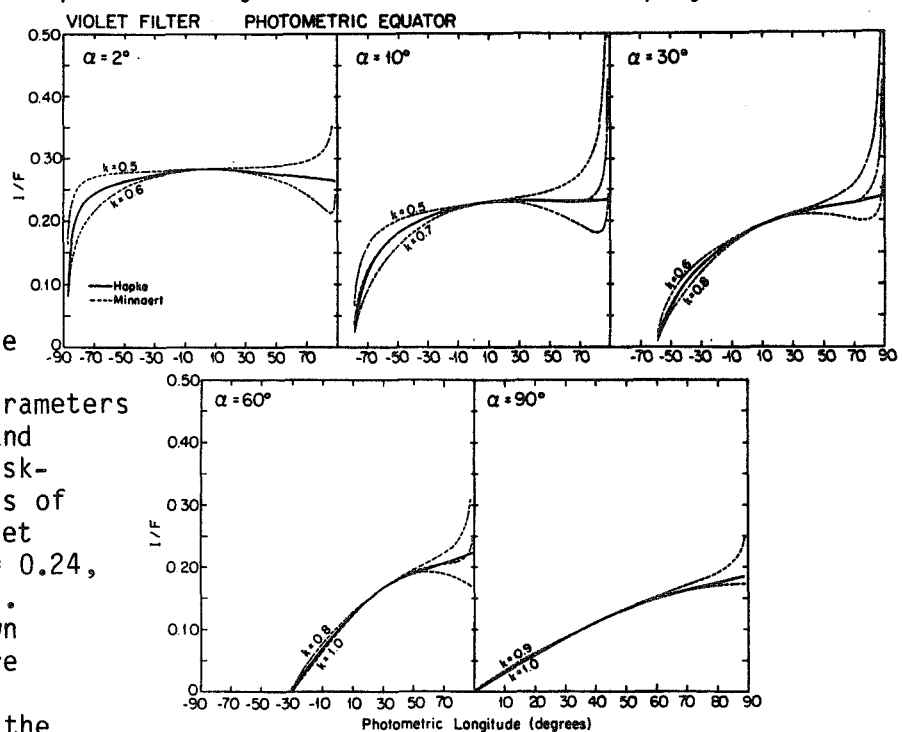


**IO: COMPARISON OF PHOTOMETRIC SCANS PRODUCED BY THE MINNAERT AND HAPKE FUNCTIONS.** Damon P. Simonelli and Joseph Veverka, Cornell University.

Experience has shown that the empirical Minnaert function is a very useful approximation to real photometric behavior near opposition (phase angle  $\alpha = 0^\circ$ ), but that in general it cannot accurately model photometric scans across the face of even a homogeneous planet at higher phase angles (see for example Goguen, 1981). Given recent work on fitting the rigorous Hapke photometric function to Voyager data for Io (Simonelli and Veverka, 1986), we can test to what degree the Minnaert function breaks down in the case of Ionian materials by comparing photometric scans produced by the two approaches.

At phase angles  $\alpha = 2, 10, 30, 60$ , and  $90^\circ$ , we have computed scans of the reflectance along the photometric equator (photometric latitude  $\Psi = 0^\circ$ ) and mirror meridian (photometric longitude  $\omega = \alpha/2$ ) that would be expected for a homogeneous planet whose surface obeys Hapke's law (Hapke, 1981, 1984). We use values of the Hapke parameters  $\tilde{\omega}_0$ ,  $h$ ,  $g$ , and  $\bar{\theta}$  derived for Io by Simonelli and Veverka (1986) in both the Voyager narrow-angle camera violet filter ( $\lambda \approx 0.42 \mu\text{m}$ ) and orange filter ( $\lambda \approx 0.59 \mu\text{m}$ ). Each calculated Hapke scan is compared with the corresponding scan predicted by Minnaert's law for various values of the Minnaert limb-darkening parameter  $k$ . For a Minnaert scan at a particular  $k$ , we arbitrarily choose the value of the reflectance parameter  $B_0$  so that the Minnaert and Hapke scans coincide at the so-called "specular point," the point where the photometric equator and mirror meridian intersect ( $\Psi = 0^\circ$ ,  $\omega = \alpha/2$ ). The violet-filter photometric scans that result from this process are shown in Figs. 1 and 2; orange-filter results are qualitatively similar and are not displayed.

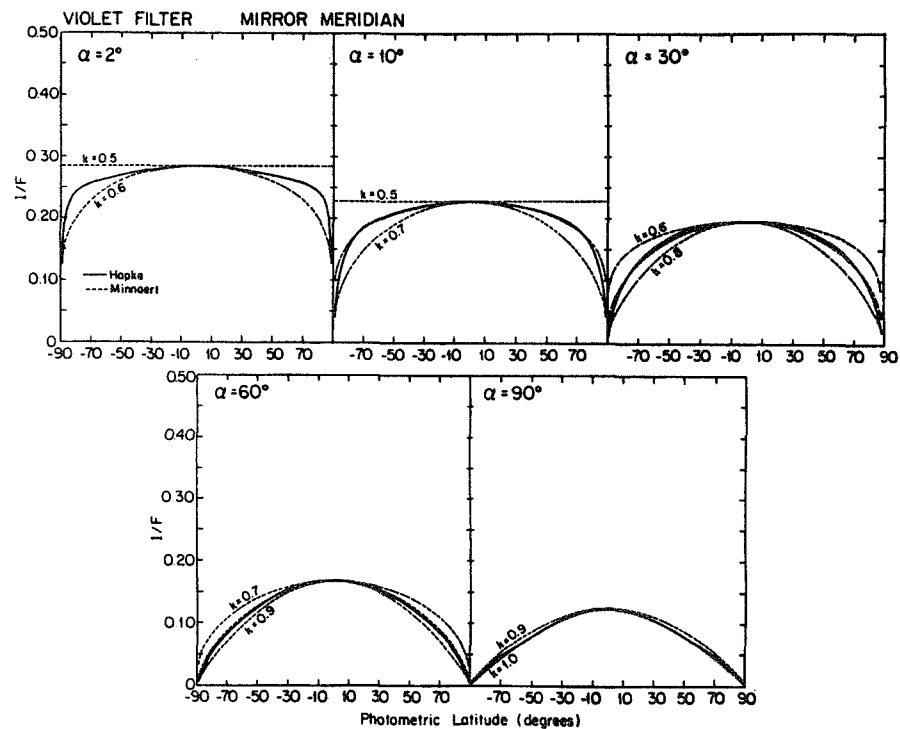
**Figure 1.** Scans at various phase angles illustrating how the reflectance  $I/F$  varies along the photometric equator. The solid curve shown at each phase angle represents a scan predicted by the Hapke function, using values of the Hapke parameters derived by Simonelli and Veverka (1986) from disk-integrated observations of Io in the Voyager violet filter:  $\tilde{\omega}_0 = 0.68$ ,  $h = 0.24$ ,  $g = -0.14$ , and  $\bar{\theta} = 25^\circ$ . The dashed curves shown at each phase angle are scans predicted by the Minnaert function for the various indicated values of the limb-darkening parameter  $k$ ; for each dashed curve, the value of  $B_0$  has been chosen so that the curve coincides with the Hapke scan at the specular point (see text).



It is apparent that the Minnaert law does not match "exactly" Hapke scans at any of the phase angles shown, even  $\alpha = 2^\circ$ . Yet our plots demonstrate that at least in the case of Io, the Minnaert law is a useful empirical tool. The Minnaert scans deviate most strongly from "real" (Hapke) behavior close to the limb of our hypothetical planet, i.e., near photometric latitude  $\pm 90^\circ$  or photometric longitude  $+90^\circ$ . However, as distances along the projected disk of a planet visible to an observer go as the sine of the photometric angles, when compiling photometric information most of the useful data come from areas within  $\approx 60^\circ$  of the sub-observer point. Figure 1 shows that along the photometric equator, the Minnaert description doesn't break down severely until  $\omega = \epsilon > 70^\circ$ , which corresponds to  $x > 0.94$ , where  $x$  is a linear scale from  $x = 0$  at the center of the projected disk to  $x = 1$  at the limb, and  $\epsilon$  is the emission angle. Thus the Minnaert function breaks down most noticeably at geometries which ordinarily contribute least to observational data sets.

We also note that in the case of Io, the Minnaert description is useful not only at small phase angles, but throughout the range of  $\alpha$  considered here. Figs. 1 and 2 suggest that the Minnaert function is as good an approximation at  $\alpha = 90^\circ$  as at  $\alpha = 2^\circ$ ; i.e., it seems able to match the Hapke scans to about the same level of approximation at all phase angles.

**Figure 2.** Same as Fig. 1, but for scans along the mirror meridian.



There is only one respect in which the Minnaert approximation worsens at high  $\alpha$ : it appears that slightly different values of  $k$  are needed to describe scans across the face of a planet in different directions. Specifically, as the phase angle increases, the value of  $k$  that best matches behavior along the photometric equator is more likely to differ

from the  $k$  that results along the mirror meridian. For example, if we compare the  $\alpha = 60^\circ$  scans for the violet filter in Figs. 1 and 2, we find  $k \approx 0.9$  along the photometric equator, but  $k \approx 0.8$  along the mirror meridian. This variation in  $k$  with scan azimuth has already been discussed by Goguen (1981) in another context. It is noteworthy that for Io the effect is not serious for moderate phase angles ( $\alpha < 60^\circ$ ).

In summary, our work with Io data indicates that the empirical Minnaert function, while not a perfect model of real photometric behavior, does provide a very useful parameterization of limb darkening at phase angles out to  $90^\circ$ , and is especially useful near opposition (cf., McEwen and Soderblom, 1984; Clancy and Danielson, 1981). Those who work with the Minnaert law, however, must bear in mind the major limitation inherent in this empirical function: Minnaert parameters are unspecified functions of the phase angle. While  $B_0(\alpha)$  for a specific material typically drops with increasing  $\alpha$ , and  $k(\alpha)$  generally increases toward higher phase angles (a trend seen in Figs. 1 and 2, as well as in Harris, 1961; Goguen, 1981; and McEwen and Soderblom, 1984), the determination of a material's  $B_0$  and  $k$  at one  $\alpha$  provides no direct information as to the values of these parameters at other phase angles.

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### References

- Clancy, R. T., and G. E. Danielson (1981). *J. Geophys. Res.* **86**, 8627-8634.  
 Goguen, J. D. (1981). Ph.D. dissertation, Cornell University, Ithaca, N.Y.  
 Hapke, B. (1981). *J. Geophys. Res.* **86**, 3039-3054.  
 Hapke, B. (1984). *Icarus* **59**, 41-59.  
 Harris, D. L. (1961). In *The Solar System III. Planets and Satellites* (G. P. Kuiper and B. M. Middlehurst, Eds.), pp. 272-342. Univ. of Chicago Press, Chicago.  
 McEwen, A. S., and L. A. Soderblom (1984). NASA Technical Memorandum 86246, pp. 261-262.  
 Simonelli, D., and J. Veverka (1986). Phase curves of materials on Io: Interpretation in Terms of Hapke's Function. *Icarus*, in press.